Four Wire Measurement

APPENDIX A

WHY SO MANY TERMINALS?

(Understanding impedance measurement techniques)

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Automatic test systems that check components "in-circuit" have reawakened interest in multi-terminal techniques for measuring impedance, initially developed for laboratory applications. State-of-the-art impedance measurement methods, incorporated in complex bridge configurations, use as many as "four-pair" (or eight-terminals) for determining a single component value.

Starting with "two-terminal: an working up to "eight-terminal" techniques, this article explores multi-terminal methods for measuring impedance, indicating types of errors and typical applications for each method. To illustrate these methods, the following discussion uses ideal voltmeters and ammeters (i.e.—the voltmeter does not draw current and the ammeter does not develop voltage). Commercial instruments using either bridge or meter techniques are not ideal and will contribute additional errors. Negligible in most applications, inaccuracies introduced by these meters can often be measured and corrected for in the equations.
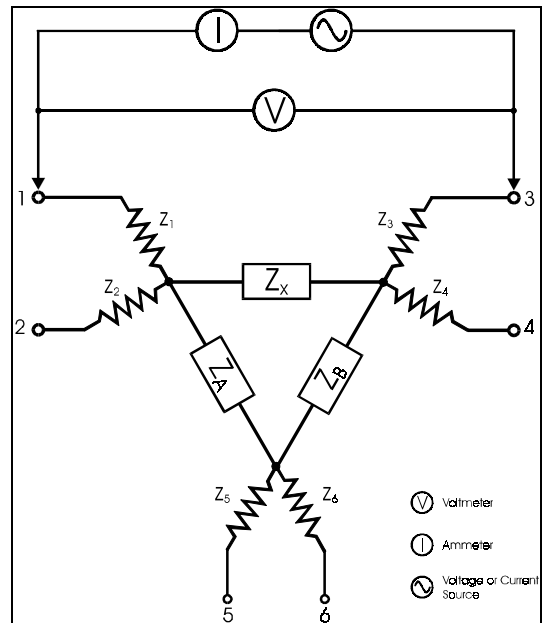
Two terminal measurements

The two-terminal technique (the simplest method) can make very accurate measurements on medium value impedances. This technique applies a current and measures a voltage and since only two terminals are available the current source and voltmeter connect to the same points, as shown in Fig. 1. This limitation introduces the "series-lead" and "shunt-load" errors. Critical when Z_x is small, the "series-lead" (z_1 and z_3) impedance adds a fixed quantity to the measured value. Important when Z_x is large, the "shunt-load" (Z_A and Z_B) impedance decreases the value of Z_M. The measured impedance becomes:

$$Z_M \cong Z_X \left(1 + \frac{Z_1 + Z_3}{Z_X} - \frac{Z_X}{Z_A + Z_B}\right)$$
$$\text{"Series-Lead"} \quad \text{"Shunt-Load"}$$

$$\cong Z_X + (Z_1 + Z_3) - \frac{Z_X{}^2}{Z_A + Z_B}$$

The "series-lead" error can be measured by shorting out the component under test and the "shunt-load" error by open-circuiting it. Incorporating these values into the above equation will give a more accurate calculation of Z_x. Shorting out the component usually present no problem, but open circuiting it becomes impractical especially when soldered into an assembly.

Fig. 1 Representing many practical situations, this circuit can serve for evaluating the various measurement techniques. Z_X is the unknown impedance, and Z_A and Z_B represent stray capacitance to the case in a three-terminal capacitor, or components of an assembly soldered in a board or chassis. The lower case z_1 through z_4 indicate the impedance introduced by leads and contacts. A terminal is counted at only one end of a wire.

For example, a two-terminal measurement of a resistor requires four connections, two at the instrument, and two on the resistor.

Three terminal measurements

Typically termed "guarded" or "direct", and excellent for evaluation high impedance, the three-terminal technique differs from the two-terminal method because it applies a voltage and measures a current, with each device connected to different points (the voltage source to pont1 and the ammeter to point 3) as shown in Fig. 2. The third terminal ("guard") serves as a common connection for input voltage and output current. Impedance from point A or B to the "guard" (point C) has no effect on the measurement, but the "series-lead" impedances (z_1, z_3 and z_5) contribute errors as shown in the following equation:

$$Z_M \cong Z_X \left(1 + \underbrace{\frac{Z_1 + Z_3}{Z_X}}_{\substack{\text{"series -} \\ \text{lead"}}} + \underbrace{\frac{Z_1}{Z_A} + \frac{Z_3}{Z_B}}_{\substack{\text{"series /} \\ \text{shung"}}} + \underbrace{\frac{Z_5 \, Z_N}{Z_A \, Z_B}}_{\substack{\text{"guard -} \\ \text{lead"}}}\right)$$

If z_1, z_3 and z_5 were zero, then $Z_M = Z_X$, eliminating any effect from Z_A or Z_B. The impedance from point 1 to 5 will load the input voltage, however the readings are still valid, since this loading does not affect the measured value, but only decreases the sensitivity of the measurement.

Adding the "guard" introduces the "series/shunt" and "guard-lead" errors. Caused by a dividing effect from z_1/Z_A and z_3/Z_B, "series/shunt" (a constant percent error) is independent of Z_x and very small at low frequencies, if Z_A and Z_B just represent stray capacitance.

Generally much smaller than the "shunt-load" error in two-terminal measurements, the "guard-lead" error becomes critical when Z_x is large. For example, in precision measurements of low-loss three-terminal capacitors using long shielded leads, capacitances Z_A and Z_B can be quite large, and any resistance in the guard connection causes a negative resistance across Z_x. The resultant negative conductance reading on a bridge could confuse the engineer making the measurement.

Many traditional four-arm bridges make fair three-terminal measurements, since they will tolerate some loading across a bridge arm.

Using a "Wagner guard" in the wheatstone bridge (Fig. 3) removes the shunt loading effects due to R_A and R_B with the auxiliary balance R_P.

Some a.c. bridges have used similar guard circuits, however, most bridges now use precision transformers (Fig.4) for accurate ratios and immunity from shunt loading. Using a dual output winding, id-loading effects reduce one output voltage, the other is reduced because of the coupling between the windings, making the effective source impedance only the winding resistance and leakage inductance, both typically very small.

Another technique for reducing shunt effects uses an inverting amplifier and feedback resistor, (Fig. 5) with the input to the amplifier at a "virtual-ground". The effective impedance to ground becomes the feedback resistance divided by the open-loop gain, resulting in very small errors.

A "double-guard" technique (Fig. 6) eliminates the "guard-lead" error by putting these impedances in series with the other lead impedances, and results in no "shunt" error, with the "guard-lead" error generally very small.

What is Impedance?

Similar to friction causing a moving object to slow down a current flowing between two pints of an electronic circuit will develop a voltage, if it encounters anything that tends to impede its free flow, such as Resistors, Capacitor and Inductors. This effect is measured in ohms in accordance with Ohm's law:

$$Z = \frac{V}{I} \quad \text{where } V = \text{voltage, } I = \text{current, } Z = \text{impedance}$$

The impedance resulting from resistors is called "resistance" (R), that from capacitors "capacitive reactance" (X_C) and that from inductors "inductive reactance" (X_L). These components of impedance are defined by the following equations

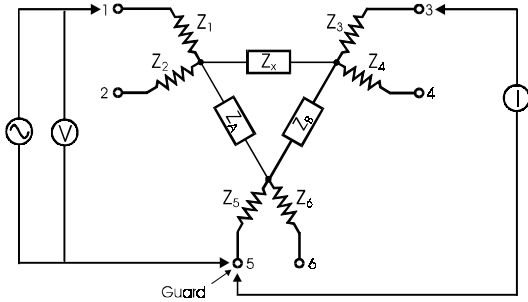**Fig. 2** Typically termed "guarded" or "direct", and excellent for evaluating high impedance, this three-terminal measuring technique eliminates the "shunt-lead" error encountered in 'two-terminal" measurements.
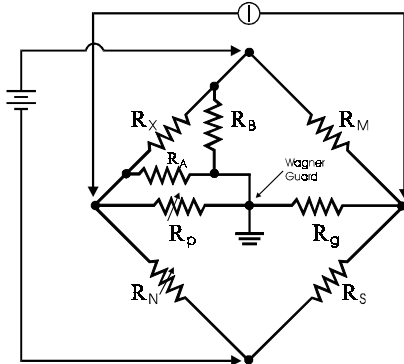

Fig. 3 Using a "Wagner guard", this three-terminal Wheatstone bridge circuit makes precise measurements by removing shunt loading effects from R_A and R_B.
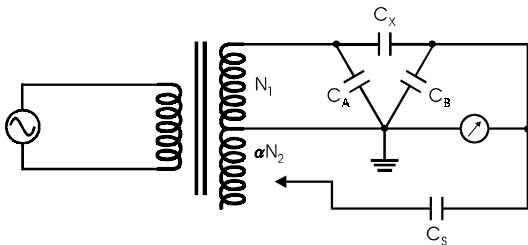

Fig. 4 Using a precision transformer, this three-terminal a.c. bridge circuit maintains an accurate voltage ratio in spite of shunt loading.
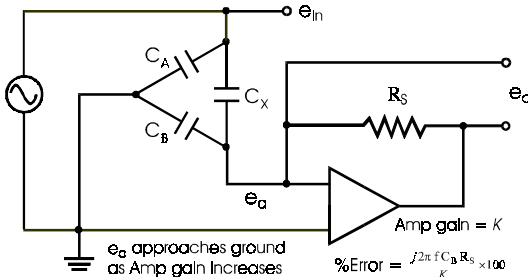

Fig. 5 With an inverting amplifier and feed-back resistor (Rs) this three-terminal impedance measurement technique places the input voltage to the amplifier (e_a) at a "virtual-ground", resulting in very small errors due to C_B.

$$R = \frac{V}{I} \quad \text{where:} \quad f = \text{frequency}$$

j = ac phase component.
C = capacitance (Farads)
L = inductance (Henry's)
R = resistance (Ohms)

$$X_C = \frac{V}{I} = \frac{1}{j\,2\pi f\,C} = \frac{-j}{2\pi f\,C}$$

$$X_L = \frac{V}{I} = j\,2\pi f\,L$$

The "double-guard" is rarely required, except in in-circuit testing when Z_A and Z_B are low values. Measured impedance with this "double-guard" becomes:

$$Z_M \cong Z_X \left(1 + \frac{Z_1 + Z_3 + Z_5 + Z_6}{Z_X} + \frac{Z_1 + Z_5}{Z_A} + \frac{Z_3 + Z_6}{Z_B}\right)$$

Four terminal measurements

Ideal for measuring low impedance, the four-terminal measurement technique (Fig. 7) applies current through one pair of leads and measures voltage on the other pair, removing the "series-lead" impedance error completely. The measured impedance, Z_M, shown in the following equation, only includes the "shunt-load" error.

$$Z_M \cong Z_X \left(1 + \frac{Z_X}{Z_A + Z_B}\right)$$
$$\text{"shunt - load"}$$

For simplicity, Fig. 7 does not display the effects of mutual inductance between leads, critical in a.c. measurements. Using coaxial cable or twisted pair leads will reduce these effects.

Used for precision dc resistance measurements, the Kelvin bridge configuration (Fig. 8) is the best known four-terminal technique. The Kelvin bridge has a main adjust (R_A) with auxiliary adjustments for "Yoke", R_a (corrects for r_3 and r_4) and "lead-adjust" R_b (corrects for r_2). The term "Kelvin-connection" is often used interchangeably with "four-terminal".

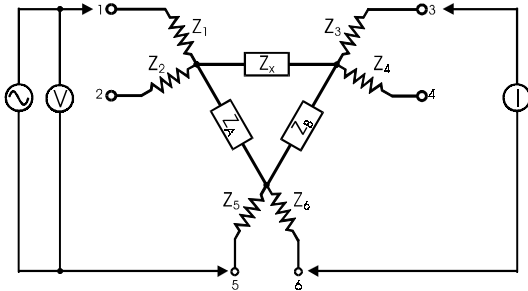**Fig. 6 Adding a "double-guard" to the three-terminal technique, this circuit eliminates the "guard-lead" error.**
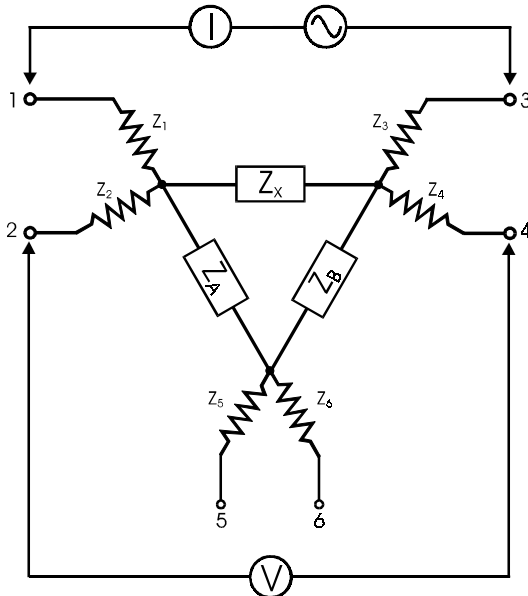

Fig. 7 For measuring low impedance, this four-terminal technique removes the "series-lead" impedance error.
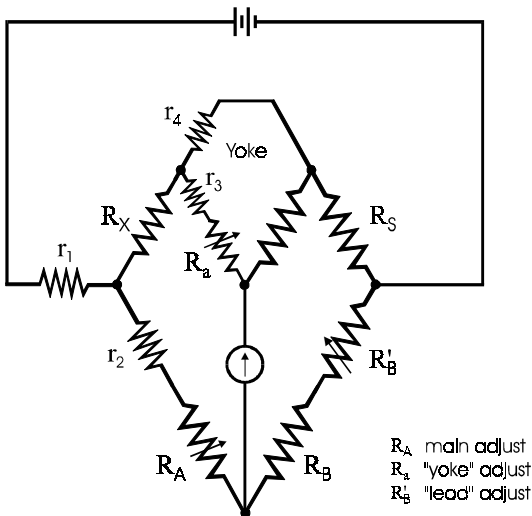

Fig. 8 Making precision four-terminal resistance measurements this "Kelvin bridge" uses three adjustments for nulling out effects from leads and contacts.

Some other four-terminal techniques include:

- The "potentiometer" method which uses voltage ratio measurements, calculating the result using the measured ratios.

- Transformers for ac measurements supply compensating voltages which reduce errors analogous to the three-terminal technique as shown in Fig. 4.

- High input-impedance differential amplifiers, which reduce "series-lead" errors.

Five terminal measurements

Capable of making precise measurements over a wide rage of impedance, the five-terminal method (Fig. 9) integrates three-terminal techniques for high impedance and four-terminal techniques for low impedance. Used primarily for remote and in-circuit evaluations this method eliminates both the "series-lead" and "shunt-load" errors found in two-terminal measurements, leaving much smaller errors caused by the interaction of series and shunt impedance. The measured impedance becomes:

$$Z_M \cong Z_X \left(1 + \frac{Z_3}{Z_B} + \frac{Z_5\, Z_X}{Z_A\, Z_B} \right)$$

$$\underset{\text{shunt"}}{\text{"series /}} \qquad \underset{\text{lead"}}{\text{"guard -}}$$

Measured Impedance

If a network contains a component with an impedance Z_X, any impedance measurement on that network will give a result Z_M in the form,

$$Z_M = \frac{Z_X(1 + A) + B}{1 + C\, Z_X}$$

where A, B, and C depend on other impedances of the network. If you make a reasonably good measurement of Z_X, then A, B and C will be small compared to unity and you can approximate the measured value as:
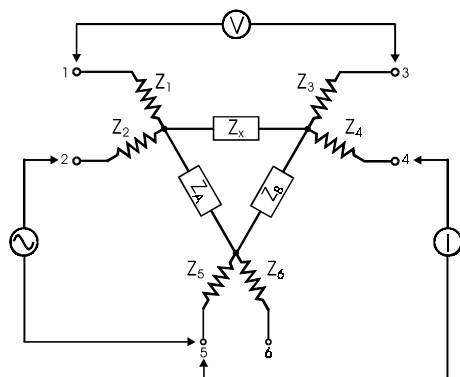
Fig. 9 Capable of making precise measurements over a wide range of impedances, this five-terminal technique integrates three and four-terminal methods, eliminating both the "series-lead" and "shunt-lead" errors.
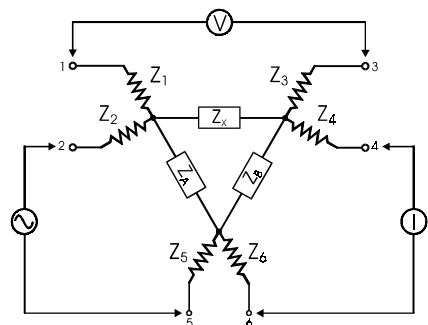
Fig. 10 With "guard" points, this six terminal method removes the "guard-lead" error encountered in five-terminal measurements.
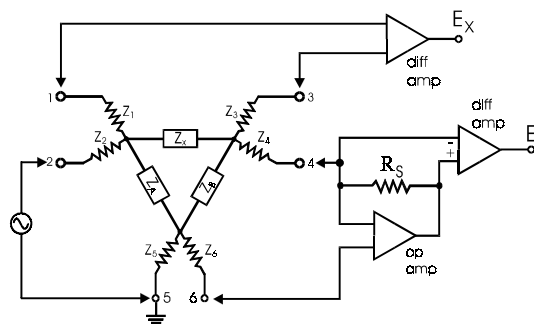
Fig. 11 Incorporating an operational amplifier and two differential amplifiers, this six-terminal circuit makes very accurate impedance measurements.
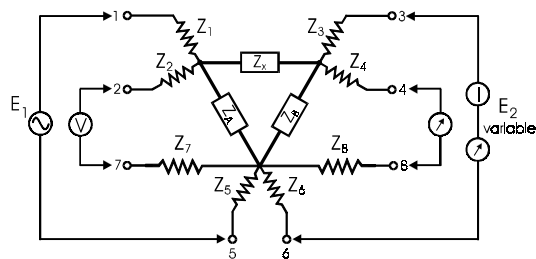
Fig. 12 Used by the National Bureau of Standards, this eight-terminal measures ZX in the network without errors.

$$Z_M = Z_X\left(1 + A + \frac{B}{Z_X} - C Z_X\right)$$

Examples in this article use this approximation for Z_M with the values for A, B and C dependent on z_1 through z_6, Z_A and Z_B.

Six terminal measurements

Eliminating the "guard-lead" errors, the six-terminal technique (Fig. 10) is an extension of the five-terminal, adding a second guard, as previously described for three-terminal measurements. Z_M for this configuration is:

$$Z_M \cong Z_X\left(1 + \frac{Z_3 + Z_6}{Z_B}\right)$$

$$\text{"series / shunt"}$$

A typical configuration for a six-terminal measurement uses an operational amplifier and two differential amplifiers (Fig.11). The addition of these active components helps reduce the effects of leads, contacts, mutual inductance and stray capacitance.

What about eight terminals?

Used by the National Bureau of Standards (NBS), the eight terminal method involves four-pairs configured to attain better than on e part in 108. (see Fig. 12) Shielded coaxial connections reduce mutual inductance between leads and prevent electrostatic/ electromagnetic pickup. The test network has no errors but shielded cables add capacitance across the meter terminals, making the measured impedance:

$$Z_M \cong Z_X\left(1 + \frac{Z_2 + Z_7}{Z_{27}} - \frac{Z_3 + Z_6}{Z_{36}}\right)$$

The errors due z2, z3 z6 and z7 are extremely small. For example, 50Ω cable (RG58/U) has about 30 pF and 11mΩ per foot, with the resulting errors being a few parts in 109 for one foot leads at 1KHz. Easily measured, this error can be corrected for, if necessary.

Impedance Measurements with Automatic Test Systems

Functional testing of loaded boards with automatic systems does not necessarily involve impedance measurement. But these automatic systems do often incorporate digital multimeters that can measure resistance and, to meet custom requirements may include impedance bridges.

On the other hand, automatic in-circuit test systems that evaluate components on loaded boards, must have multi-terminal impedance measurement capability to isolate the component under test from the effects of adjacent circuitry. Most of these systems include the three-terminal technique, using the third terminal as a "guard" point to place selected nodes at an equipotential, thereby removing the effects of components connected between these nodes. This method is adequate for detecting manufacturing faults and wrong components. Some firms now use four, five and even six-terminal techniques for increased accuracy. Future equipment designs could conceivably involve widespread use of six-terminal techniques, but the added wiring and switching would substantially increase equipment costs.

A thorough description of these in-circuits testers will be presented as part of a feature article on automatic test systems in the March 1979 issue of *Electronics Test*.

Conclusions

The most difficult impedance measurements are those on components "in-situ", that is, soldered in a network with other components. It's easy to imagine values or combinations of components in the circuit of Fig. 1, which would make it impossible to measure Z_X accurately with available instrumentation. However, today's techniques and instruments can measure a good percentage of the components on a typical assembly.

For instance if $Z_X = 10K\Omega$ and $z_5 = 1m\Omega$ in the circuit of Fig. 2 the "guard-lead" error becomes:

$$\frac{Z_5 \, Z_X}{Z_A \, Z_B} = \frac{(0.001)(10.000)}{(10)(10)} = 0.1$$

This causes a 10% error in the measured value Z_x. Increasing demand for more accurate in-circuit tests could result in wide spread use and better implementation of six-terminal impedance measurement techniques in automatic systems.

Now a senior staff scientist, Henry Hall has designed several of Gen Rad's impedance measuring instruments and standards. In his 30 years at Gen Rad, he has served as an Engineer, a leader of the Low Frequency Impedance Measurements group, Engineering Staff consultant and Senior Principle Engineer. A member of Phi Beta Kappa, Tau Beta Pi, Eta Kappa Nu, Sigma Xi and a Fellow of the IEEE, he received a B.A degree from Williams College, and a B.S. and M.S. degrees in electrical engineering form the Massachusetts Institute of Technology.

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