

Understanding "Walking the Beam"

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Introduction

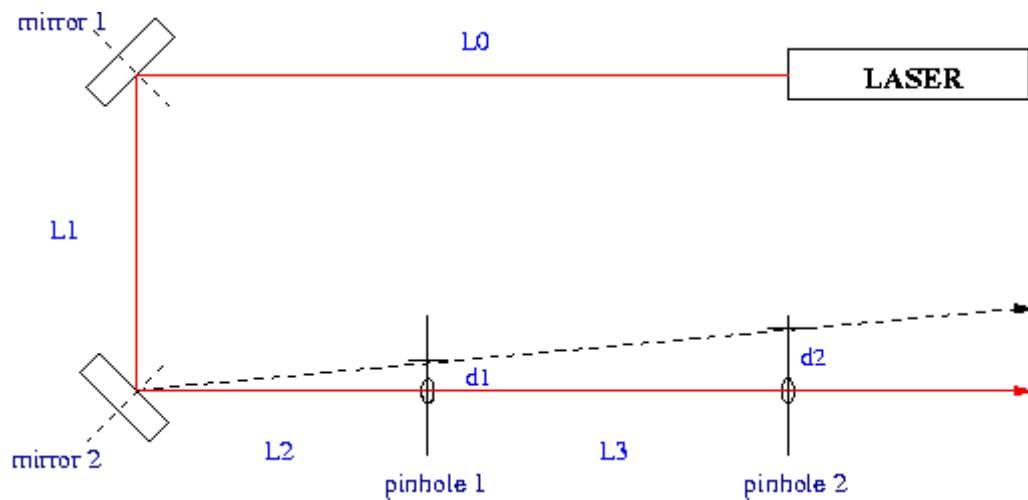
"Walking the beam" is the process of aligning a laser beam using two adjustable mirrors in such a way that it will reach a specific point in space with a specific angle. Since laser beams are straight lines, this is equivalent to making the beam pass through two consecutive apertures. This process is used to make two laser beams precisely collinear when aligning an interferometer, for example, or when coupling light into an optical fiber.

It can be a difficult and frustrating task to properly adjust the four knobs that control the horizontal and vertical angles of the mirrors. Without making the correct sequence of adjustments it is even possible to move further away from the goal instead of towards it! Fortunately there is a specific procedure that can be followed that is sure to work eventually. The procedure involves considering the two mirrors as having (partly) separate functions. The first mirror seen by the beam is called the *position mirror*; it primarily affects the position of the beam at the first aperture. The second mirror is the *angle mirror*; it primarily affects the angle of the beam at the first aperture.

The purpose of this project was to analyze mathematically the systematic process of steering a beam through two pinholes. The equations derived to describe the algorithm were graphed to provide a visual interpretation of what it means to "walk the beam."

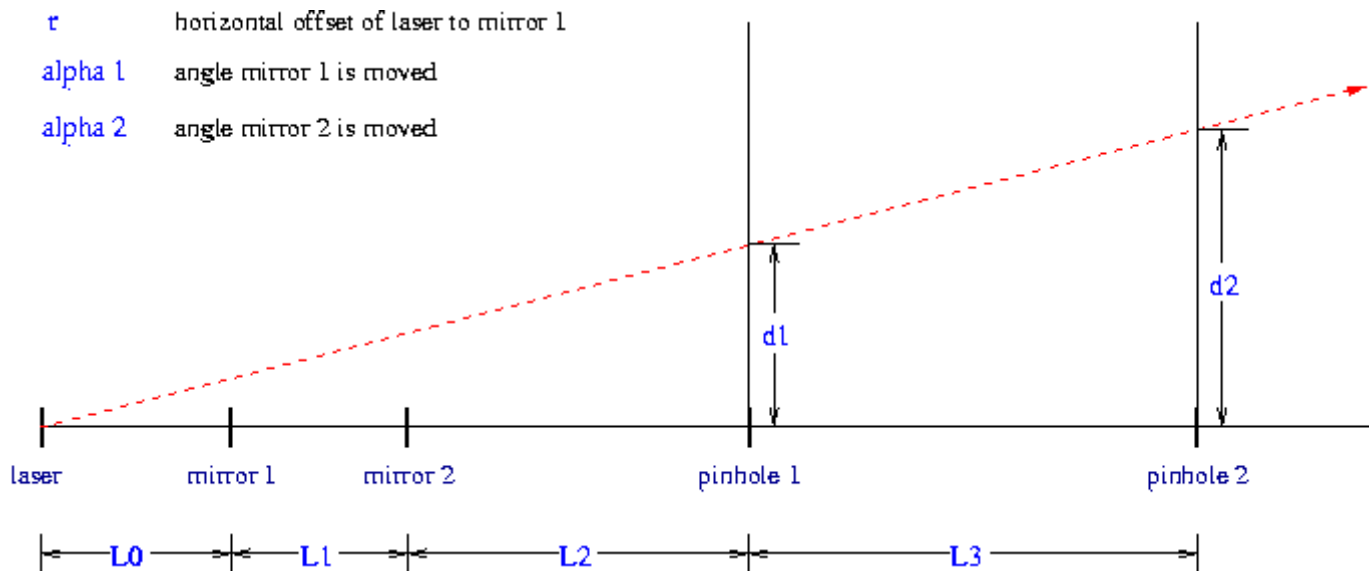
Setup

The setup used in this project consisted of a HeNe laser emitting a beam that was reflected off mirror 1 to mirror 2. The beam passed through two pinholes. The pinholes were irises that could be opened to so that the misaligned beam was able to pass. The distances between the components ranged from a few centimeters to a meter.



Line Diagram

To more easily represent the setup, it can be drawn as a straight line. This is because the distances between the optics components are not changed. The misaligned beam can be drawn at an angle from the laser. When the beam is reflected off mirror 1, its trajectory is altered by the addition of the angle of reflection. The angle is called alpha 1. The same is true for the beam at mirror 2, and the angle is called alpha 2. When the beam is misaligned, the distance the beam strays from the center of the pinhole is d_1 . The distance that the beam strays from the center of pinhole 2 is labelled d_2 .

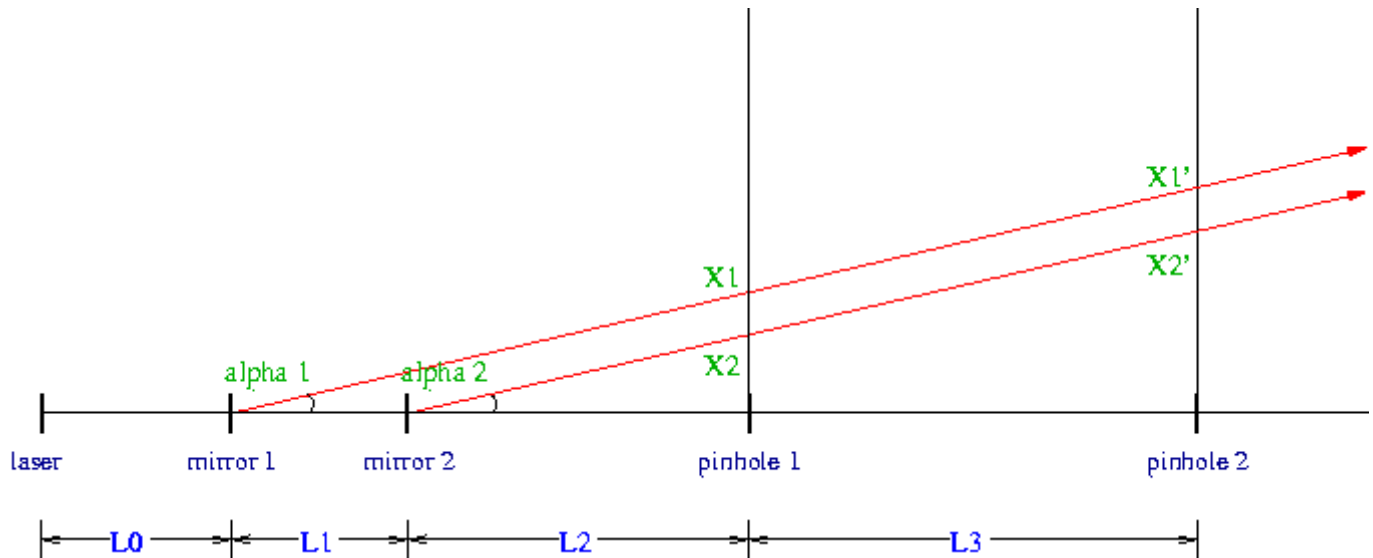


In this setup, only the offset along the x-axis was measured and calculated because the procedure is the same for the beam along the y-axis. Since the location of the beam in the two axes is independent of each other, it was only necessary to consider one axis.

Mirrors

Using the line schematic, the purposes of a position and angle mirror are clearly delineated. Mirror 1, the position mirror of this setup, moved the beam a greater d_1 (here represented as x_1) than mirror 2 for the same angle, α . Mirror 2, the angle mirror, moves the beam a lesser

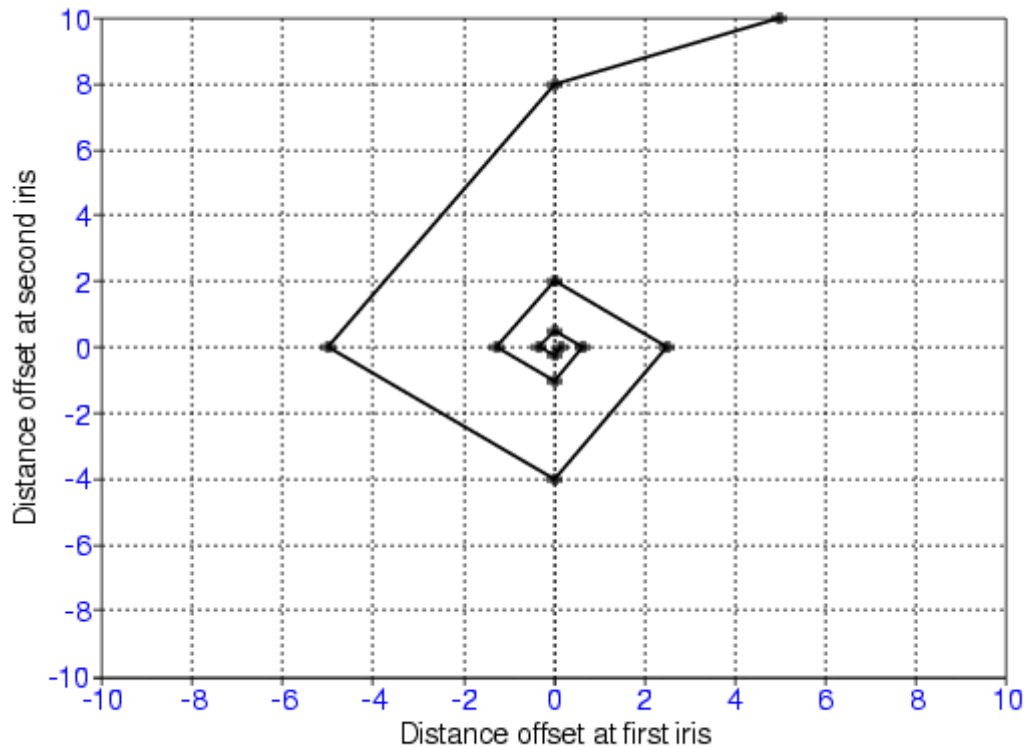
distance than mirror 1 for the same α . However, it moves the beam a greater distance from pinhole 2 (x_2') than it does from pinhole 1 (x_2) for the same angle. Therefore, it is more effective for adjusting the offset of the beam at the second pinhole. While this is true for mirror 1 as well, it is more pronounced in mirror 2 because mirror 2 cannot move the beam as far as mirror 1 along pinhole 1 for the same angle.



In practice, this requires mirror 1 to be used to adjust the location of the beam along pinhole 1 while mirror 2 is used to adjust the beam's location along pinhole 2. In the first iteration, mirror 1 is adjusted so that $d_1 = 0$. Therefore, d_2 must not be 0 (or the beam will already be aligned). In the "half" iteration, mirror 2 is adjusted so that $d_2 = 0$. d_1 no longer is 0. In the second iteration, d_1 is set again to 0. d_2 should now have decreased as compared to d_2 in iteration 1.

Hypothetical Plot

Following the method outlined above, it was originally thought that the offset distances of the beam from iteration to iteration could be plotted as a spiral. A hypothetical spiral plot was created using the spreadsheet program Quattro Pro:



Plotted along the x-axis is d_1 and along the y-axis is d_2 . Before aligning, the offset from both pinholes is an arbitrary value. Mirror 1 is moved so that $d_1 = 0$ and the first value is plotted along the y-axis only (d_2 does not equal 0). For the half iteration, $d_2 = 0$, and the second value is plotted along the x-axis only (d_1 does not equal 0).

However, after performing the procedure, it was determined that the spiral plot was wrong. For a spiral to be plotted from the data, the offset distances must alternate on which side of the pinhole they are found (positive and negative x-axis). This does not occur because the beam is never moved beyond its "target" (i.e. the pinhole). Instead, the graph appears a zig-zig since the offset distances always remain either in the positive or the negative x-axis and y-axis; the plot is located in one quadrant only.

Iterations

To derive the equations, the initial coordinates of the beam were (r, θ) where r and θ are the initial horizontal displacement and displacement angle of the laser from the first mirror, respectively. The beam was then reflected off mirror one, translated to mirror 2, reflected off mirror 2, and translated through pinholes 1 and 2. The final coordinates of the beam were $r + L_1\theta + (L_2+L_3)(\theta + \alpha_1 + \alpha_2)$, $\theta + \alpha_1 + \alpha_2$.*** This is the distance, d_2 from pinhole 2. The coordinates at d_1 were $r + L_1\theta + (L_2)(\theta + \alpha_1 + \alpha_2)$, $\theta + \alpha_1 + \alpha_2$.*** Using the values for L_1 , L_2 , L_3 , r_{init} , and θ_{init} , d_1 and d_2 were calculated. Then, d_1 was set to 0 and α_1 and d_2 were calculated. d_1 was the offset of the first iteration. For the half iteration, $d_2 = 0$ and α_2 was solved for. d_1 was then able to be calculated, the offset of the first half iteration. To calculate d_2 for the second iteration, d_1 was set again to 0 and the procedure was repeated.

Step	Action	Formula
1	Pick initial r, θ	
2	Solve for d_1	1
3	Solve for d_2	2
4	Adjust α_1 for $d_1 = 0$	3
5	Solve for d_2	2
6	Adjust α_2 for $d_2 = 0$	4
7	Solve for d_1	1
8	Go To 4	

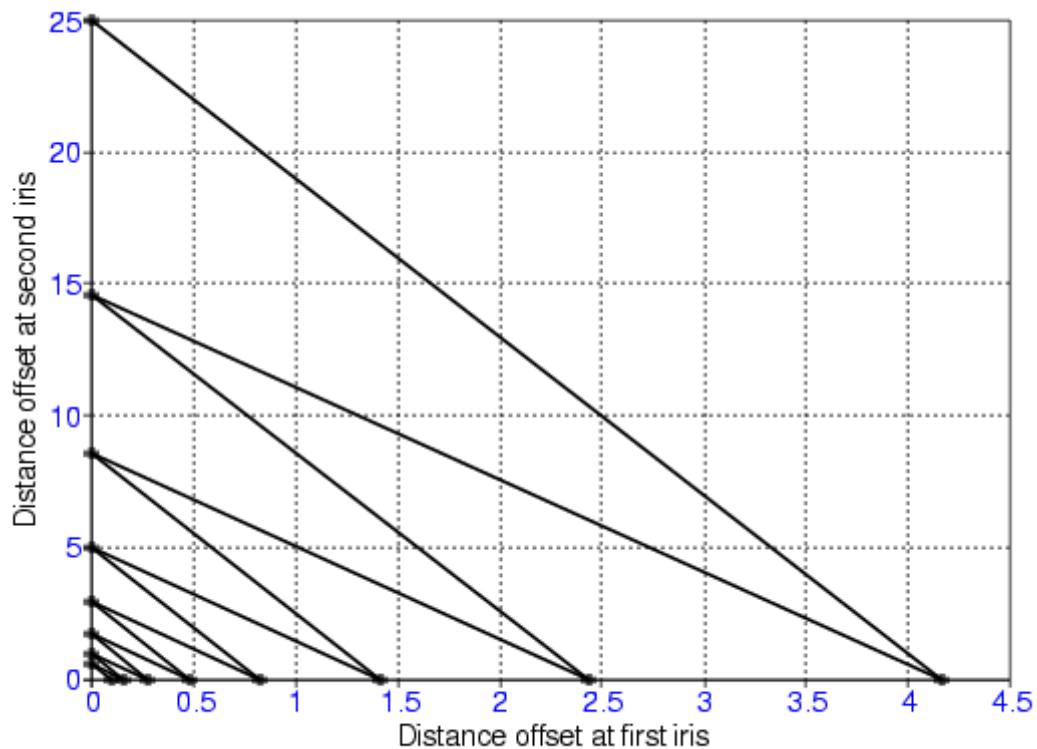
Formula 1: $d_1 = r + L_1(\theta + \alpha_1) + L_2(\theta + \alpha_1 + \alpha_2)$

Formula 2: $d_2 = r + L_1(\theta + \alpha_1) + (L_2 + L_3)(\theta + \alpha_1 + \alpha_2)$

Formula 3: $\alpha_1 = (-r - \theta(L_1 + L_2) - \alpha_2 L_2) / (L_1 + L_2)$

Formula 4: $\alpha_2 = (-r - \theta(L_1 + L_2 + L_3) - \alpha_1(L_1 + L_2 + L_3)) / (L_2 + L_3)$

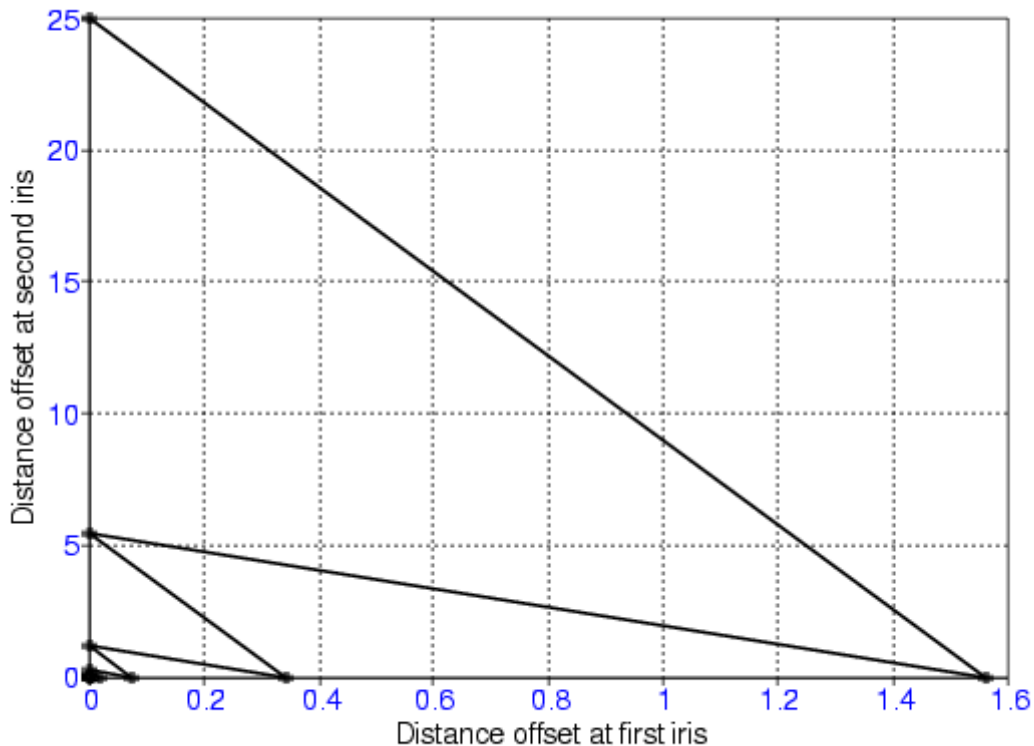
The equations were inserted into a Quattro Pro spreadsheet to calculate the offset distances. When $L_1 = 0.6$, $L_2 = 0.6$, $L_3 = 3$, $r_{\text{init.}} = 0.1$, and $\theta_{\text{init.}} = 0.1$, the convergence plot was:



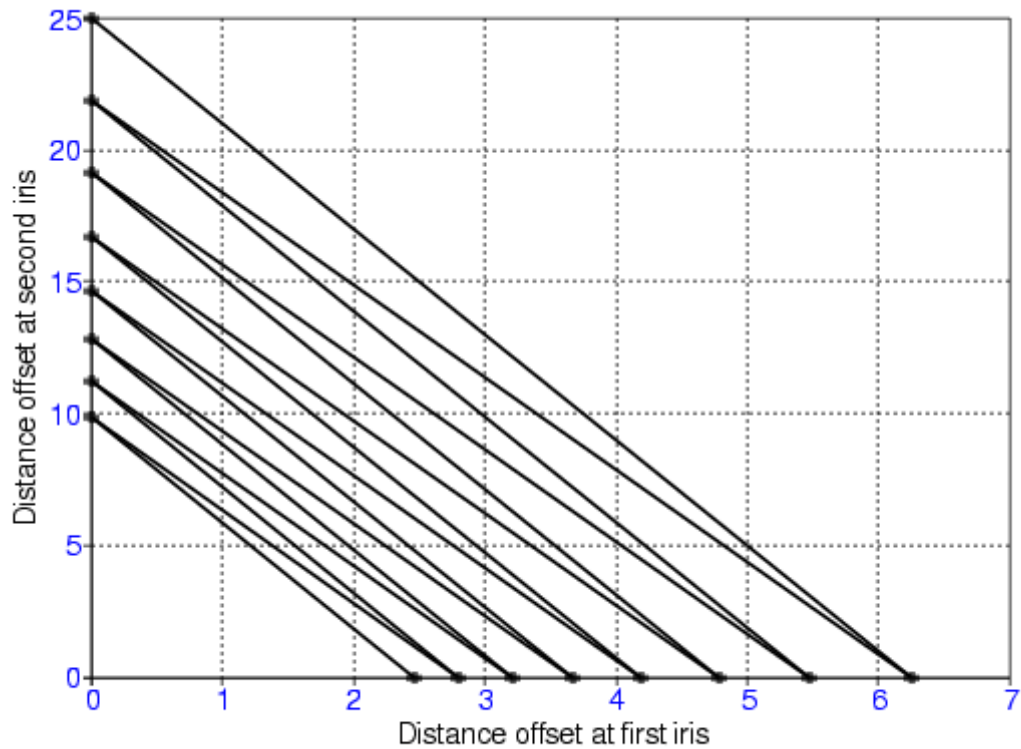
Convergence

The rate of convergence was found to be related only to distances $L1$ and $L2$ of the setup. When $L1$ is greater than $L2$, it serves as a better position mirror because the difference in the offsets of the beam from pinhole 1 when mirror 1 and mirror 2 are set at the same angle is $\alpha L1^{***}$. When $L1$ is nearly 0, the difference in the offsets is nearly 0, and mirror 1 does not act as an effective position beam. Conversely, when $L2$ is nearly 0, it acts as a better angle mirror because it moves the beam farther from pinhole 2 than it does from pinhole 1. The rate of convergence increases since moving mirror 2 to set $d2 = 0$ does not significantly move the beam across pinhole 1.

The rates of convergence were simulated in Quattro Pro using the derived equations. The plot below exhibits a fast rate of convergence. For the plot, $L1 = 1$, $L2 = 0.2$, $L = 3$, $r\text{-initial} = 0.01$, and $\theta\text{-initial} = 0.01$. $L1$ is greater than $L2$ by a factor of five.

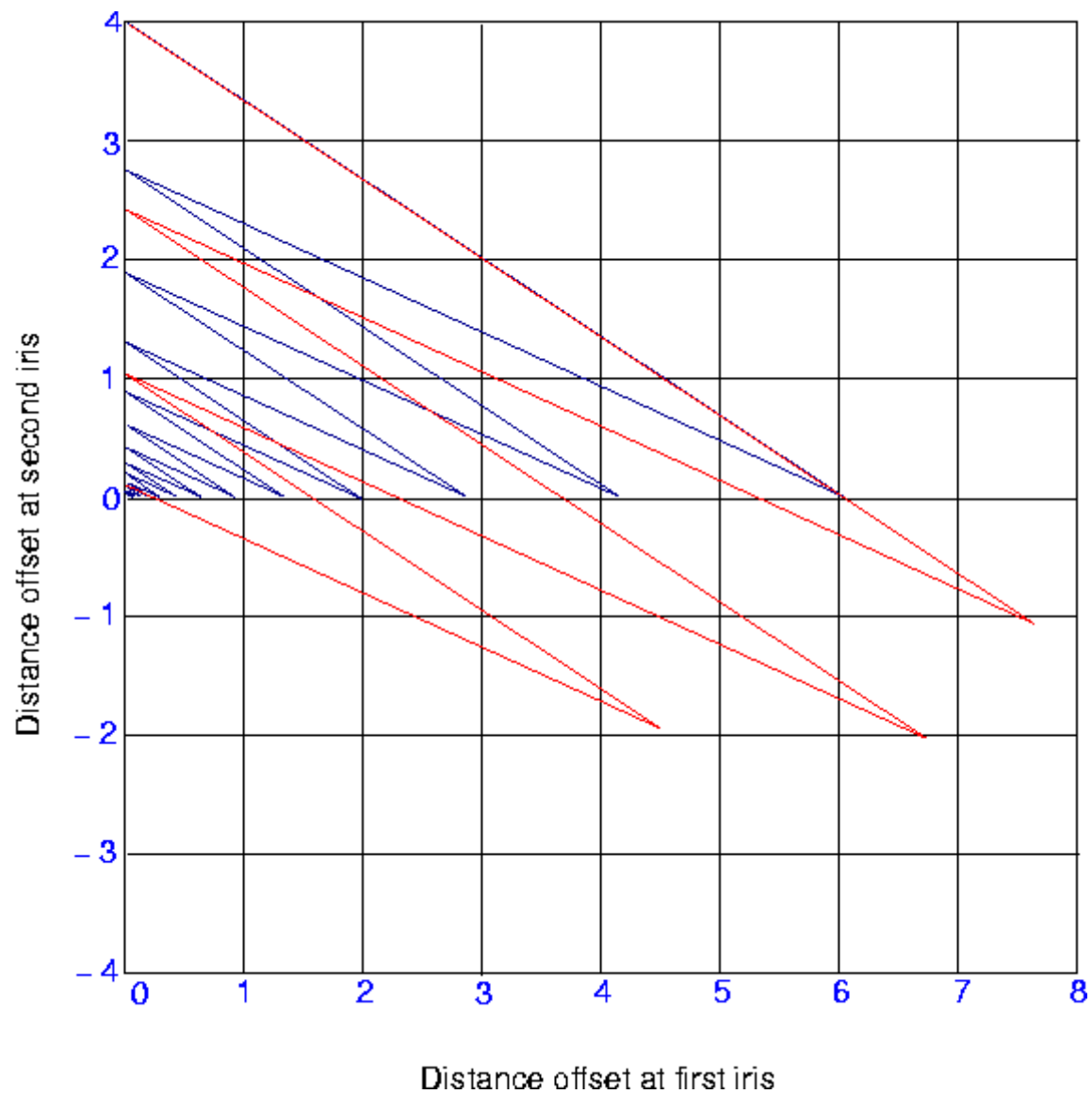


For the next plot, $L1 = 0.2$, $L2 = 1$. All other dimensions are kept constant. $L2$ is greater than $L1$ by a factor of five. The rate of convergence is slow.



Overshooting

With the procedure in this report, it is impossible to perfectly align the beam. D1 and d2 will never equal 0 at the same time unless one overshoots the "target" and moves the beam past the pinhole. Graphically, this is explained by extending the line past the d1 axis (past the second pinhole). The next line is drawn parallel to the line that would have been drawn from the d1 axis to the d2 axis. The offset along the d2 axis is less for the overshoot plot than it is for the original plot. Therefore, the rate of convergence is faster when the beam passes the pinhole during alignment.



Conclusion

In this project, the equations for laser alignment through two pinholes were derived. The beam's convergence was analyzed and then related to the concept of a position and angle mirror. Using the results of this report, the rate of convergence can be maximized to align the beam in the least number of steps.